

**Department of Electronics & Communication Engineering.  
Bundelkhand Institute of Engineering & Technology, Jhansi.**

Assignment Sheet 2  
Information Theory and Coding (DC 13)

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Due Date : November 07, 2011

Problems : 10

1. State and prove "Kraft Inequality" with the help of appropriate example.
2. Prove the following :  

$$H(X, Y) = H(X/Y) + H(Y)$$

$$= H(Y/X) + H(X)$$
3. Let  $a_1, a_2, \dots, a_k$  be a set of disjoint events and let  $P(a_1), \dots, P(a_k)$  and  $Q(a_1), \dots, Q(a_k)$  be two different probability assignment

$$(\sum P(a_k) = \sum Q(a_k) = 1)$$

Prove the following statements.

$$\sum P(a_k) \log\left(\frac{P(a_k)}{Q(a_k)}\right) \geq 0$$

$$\sum \left(\frac{P(a_k)}{Q(a_k)}\right)^2 \geq 1$$

4. A discrete memory less source emits one of the four symbols  $S_1, S_2, S_3$  &  $S_4$  with the probabilities  $\frac{1}{3}, \frac{1}{6}, \frac{1}{4}$  &  $\frac{1}{4}$  respectively. The successive symbols emitted by the source are statistically independent. Find
  - (A) Amount of information gained by observing the source emitted each of these symbol.
  - (B) Entropy of the source
  - (C) If the source alphabet consist K equiprobable symbols, explain why the use of a fixed length code for the representation of such a source is about as efficient as code can be.
  - (D) For part© , what condition have to be satisfied by K and the code length for the code efficiency to be 100%.
5. Give the mathematical explanation of the first fundamental source coding theorem.
6. Explain Shanon fano and Huffman coding theorems.
7. Prove that mutual information channel is symmetric. Also write down the properties of mutual information.
8. Explain entropy with its properties. Also give the mathematical expressions of the same.

9. Prove the below relationship of mutual information.

(A)  $I(X ; Y) = H(X) - H(X/Y)$

(B)  $I(X ; Y) = H(Y) - H(Y/X)$

10. Prove the following relationship.

$$I(X ; Y) = H(X) + H(Y) - H(X ; Y)$$