Bundelkhand Institute Of Engineering & Technology, Jhansi

Information Theory & Coding (DC 13)
Assignment Sheet 01

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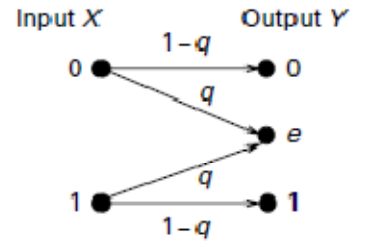
Due Date :

1. A source emits one of the four possible symbols' during each signaling intervals. The symbols occur with the probabilities
$P_0 = 0.4$
$P_1 = 0.3$
$P_2 = 0.2$
$P_3 = 0.1$
Fine Amount of information gained by observing the source emitting each of these symbols.

2. Consider a discrete memoryless



Channel transition probabilities
$p(y|x)$

Show that channel capacity of binary ensure channel with erasure probability q as shown in the above figure is given by (1-q).

3.

Consider a discrete memoryless source with source alphabet $\mathcal{S} = \{s_0, s_1, \ldots, s_{K-1}\}$ and source statistics $\{p_0, p_1, \ldots, p_{K-1}\}$. The nth extension of this source is another discrete memoryless source with source alphabet $\mathcal{S}^n = \{\sigma_0, \sigma_1, \ldots, \sigma_{M-1}\}$, where $M = K^n$. Let $P(\sigma_i)$ denote the probability of σ_i.

(a) Show that

$$\sum_{i=0}^{M-1} P(\sigma_i) = 1$$

which is to be expected.

(b) Show that

$$\sum_{i=0}^{M-1} P(\sigma_i) \log_2\left(\frac{1}{p_{i_k}}\right) = H(\mathcal{S}), \qquad k = 1, 2, \ldots, n$$

where p_{i_k} is the probability of symbol s_{i_k}, and $H(\mathcal{S})$ is the entropy of the original source.

(c) Hence, show that

$$\begin{aligned} H(\mathcal{S}^n) &= \sum_{i=0}^{M-1} P(\sigma_i) \log_2 \frac{1}{P(\sigma_i)} \\ &= nH(\mathcal{S}) \end{aligned}$$

4.

The *binary erasure channel* has two inputs and three outputs as described in Figure P9.23. The inputs are labeled 0 and 1, and the outputs are labeled 0, 1, and e. A fraction α of the incoming bits are erased by the channel. Find the capacity of the channel.

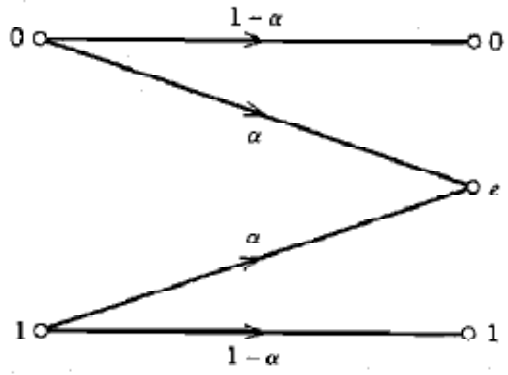


FIGURE P9.23

5.

A voice-grade channel of the telephone network has a bandwidth of 3.4 kHz.

(a) Calculate the information capacity of the telephone channel for a signal-to-noise ratio of 30 dB.

(b) Calculate the minimum signal-to-noise ratio required to support information transmission through the telephone channel at the rate of 9,600 b/s.

6.

Alphanumeric data are entered into a computer from a remote terminal through a voice-grade telephone channel. The channel has a bandwidth of 3.4 kHz and output signal-to-noise ratio of 20 dB. The terminal has a total of 128 symbols. Assume that the symbols are equiprobable and the successive transmissions are statistically independent.

(a) Calculate the information capacity of the channel.

(b) Calculate the maximum symbol rate for which error-free transmission over the channel is possible.

7.

Consider a binary symmetric channel characterized by the transition probability p. Plot the mutual information of the channel as a function of p_1, the *a priori* probability of symbol 1 at the channel input; do your calculations for the transition probability $p = 0$, 0.1, 0.2, 0.3, 0.5.

8.

A source emits one of four symbols s_0, s_1, s_2, and s_3 with probabilities 1/3, 1/6, 1/4, and 1/4, respectively. The successive symbols emitted by the source are statistically independent. Calculate the entropy of the source.

9. Write down the properties of the mutual information and give the mathematical expression.

10. Describe Morkov model for information source with one suitable example.